First to call 50: winning strategy using backward induction

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1 Problem statement

Setup. Two players take turns calling out integers. The first person to call out 50 wins.
Rules:

- The first player must call an integer between 1 and 10 (inclusive).
- Each subsequent number must be strictly greater than the previous number by at least 1 and at most 10.
- The player who first calls exactly 50 wins immediately.

Question: Should you go first, and what is the optimal strategy?

2 Informal analysis

Let the calls during the game be denoted $C_1, C_2, \ldots, C_w$, where C_1 is the first call and $C_w = 50$ is the winning call. The rules imply

$$1 \le C_1 \le 10, \qquad 1 \le C_i - C_{i-1} \le 10 \quad \text{for } i \ge 2.$$

We analyze the game by backward induction: identify positions (numbers) that are *winning positions* for the player about to move, meaning that from such a number the player to move can force a win with perfect play.

3 Backward induction

To call 50 on your turn, the previous call must lie in the range $[40, 49]$. Therefore, if you can force the opponent to hand you any number in $[40, 49]$, you will win by calling 50.

To force the opponent into $[40, 49]$, you would like to hand them the number 39, because from 39 the opponent's allowed responses are $[40, 49]$ and then you can call 50.

Repeating this backward:

$$39,\ 28,\ 17,\ 6$$

are the critical numbers: if you call one of these on your turn, the opponent is forced into an interval that allows you to move to the next critical number (adding 11 each time), eventually reaching 39 and then 50.

Thus the numbers

$$6,\ 17,\ 28,\ 39,\ 50$$

form the thread of winning positions spaced 11 apart.

4 Formal justification

Claim. If a player on their turn calls a number congruent to $6 \pmod{11}$ (i.e., of the form $6 + 11k$), then with perfect play that player can force a win; conversely, if the player to move holds a number not congruent to $6 \pmod{11}$, the opponent can (by proper play) move to a number congruent to $6 \pmod{11}$.

Proof. Suppose on your turn you call $n = 6 + 11k$ for some integer $k \geq 0$ (so $n \leq 50$ as relevant). The opponent must respond with a number m satisfying

$$n + 1 \leq m \leq n + 10.$$

All such m satisfy $m \equiv n + r \pmod{11}$ for some $r \in \{1, \dots, 10\}$, so $m \not\equiv 6 \pmod{11}$. In particular, no response of the opponent can be congruent to $6 \pmod{11}$. Therefore, you can always respond by calling

$$n' = n + 11 = 6 + 11(k + 1),$$

which is legal because $n' - m \leq n' - (n + 1) = 10$ and $n' - m \geq n' - (n + 10) = 1$. Thus you move to the next number in the sequence $6, 17, 28, 39, 50$.

This induction continues until you call 50. Hence control of numbers congruent to $6 \pmod{11}$ lets you force the win.

Conversely, if it is your opponent's turn and they hold a number not congruent to $6 \pmod{11}$, you can always choose a response that lands on the next $6 \pmod{11}$ number, so the other player can be forced back into the losing class.

Therefore the positions congruent to $6 \pmod{11}$ are precisely the winning positions for the player about to move.

5 Concrete strategy

Strategy. Go first and call 6. After that, whenever your opponent calls some number, respond by playing the unique number that brings the running total to the next term of the sequence

$$6,\ 17,\ 28,\ 39,\ 50,$$

i.e., always move to the smallest number of the form $6 + 11k$ that is at most 50. This guarantees that you will be the one to call 50.

6 Winning sequence (example)

Turn	Player	My call	Opponent's possible responses	Notes
1	Me	6	—	start at a $6 \pmod{11}$ position
2	Opponent	—	[7, 16]	any call in this interval
3	Me	17	—	move to next $6 \pmod{11}$ number
4	Opponent	—	[18, 27]	
5	Me	28	—	
6	Opponent	—	[29, 38]	
7	Me	39	—	
8	Opponent	—	[40, 49]	
9	Me	50	—	win

7 Why the strategy works

Two simple facts suffice:

1. On any move you can increase the current number by 1–10.
2. The residue classes modulo 11 partition the integers into blocks of size 11. By forcing the running total into the residue class $6 \pmod{11}$ on your turns, you deny the opponent the ability to do so on theirs; therefore you can maintain control and eventually reach 50.

8 Conclusion

Summary. The first player wins with perfect play by starting at 6 and then repeatedly adding 11 (after the opponent moves), following the sequence

$$6,\ 17,\ 28,\ 39,\ 50.$$

Thus the winning positions are exactly those congruent to $6 \pmod{11}$.

Remark: generalization. The same backward-induction method generalizes: if on each move a player may add between 1 and M, and the target is T, then winning positions are those congruent to $r \pmod{M+1}$ where r is the smallest positive integer such that $T \equiv r \pmod{M+1}$ and that is reachable from a legal initial move.